**The comparison test**

**Theorem:** (The comparison test)

Let ,  and  be series with nonnegative terms. Suppose that for some integral 

 for all .

(i) If  converges, then  also converges.

(ii) If  diverges, then  also diverges.

**Example:**

Which of the series converge, and which diverge

(1)  (2)  (3) 

**Solution:**

(1) 



 diverges because  (p-series), then  diverges.

(2) 



 diverges because  (p-series), then  diverges.

(3) 



 converges because  (p-series), then  converges.

**Theorem:** (Limit comparison test)

Suppose that  and  for all ( an integer).

1. If , then  and  both converge or both diverge.
2. If , and converges, then  converges.
3. If , and diverges, then  diverges.

**Example:**

Which of the following series converge, and which diverge

(1)  (2)  (3) 

(4)  (5)  (6) 

**Solution:**

(1) 

. Consider .

The series  is divergent series because  (p-series).



 is divergent, then is divergent.

(2) 

. Consider .

The series  is convergent series because  (p-series).



 is convergent, then is convergent.

(3) 

. Consider .

The series  is convergent series because  (p-series).



 is convergent, then is convergent.

(4) 

. Consider .

The series  (geometric series) is convergent series because .



 is convergent, then is convergent.

(5) 

. Consider .

The series  is divergent series because  (p-series).



 is divergent, then is divergent.

(6) 

. Consider .

The series  (geometric series) is convergent series because .



 is convergent, then is convergent.

**Exercises**

Which of the following series converge, and which diverge

     

 